This paper was published in *Imre Lakatos and Theories of Scientific Change*, edited by K. Gavroglu, Y. Goudaroudis, Nicolacopoulos, 284 - 304, Kluwer Academic Publishers.

ON INCOMMENSURABILITY[1]

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It is my aim in this paper to introduce a precise core for a theory of incommensurability - a 'core' as contrasted to a pure definition or an explication in the sense of logical empiricism. The picture of a theory I have in mind here is that of structuralist meta-theory[2] according to which a theory consists of a basic core which covers the phenomena common to all applications of the theory, and which is empirically rather empty, together with various specializations of this core which are valid only in special subsets of intended applications, and which usually have respectable empirical content. It turns out that the basic requirements characterizing this core are not thrillingly new. In fact, they were stated in some form quite some time ago by Paul Feyerabend[3] when he described incommensurability as logical inconsistency. The way we are led to these requirements, however, starts from a basic intuition which may be traced back at least to Thomas Kuhn's PSA paper[4] where incommensurability in one place is characterized as overall structural change in the light of a stable taxonomy.

Since the reasoning towards the conditions for incommensurability here is quite independent of former writings the coincidence of these conditions may be regarded as a piece of confirmatory evidence for them. Moreover, by stressing the character of a theory, we insist on the one hand on taking seriously the concrete historical examples. On the other

[1]This paper was written under DFG-project Ba 678/3-1. I am indebted to George Berger for critical suggestions on an earlier draft as well as for correcting my English.

[2]See, for instance, (Balzer, Moulines, Sneed, 1987), in particular Chap. IV.

[3]See, for instance, (Feyerabend, 1965, 1970). Compare also (Scheibe, 1976) on this point.

[4]Kuhn (1983).

hand we are free in adjusting the conceptual model to fit these phenomena. Finally, this theory contributes to the study of intertheoretic relations and therefore also to that of scientific progress and rationality.

1. THE BASIC PHENOMENA

The phenomena on which discussions about incommensurability are based are given in the form of periods in the historical development of a science in which one theory is replaced by another one such that there is much controversy about whether the new theory is better than the old one and should be accepted instead of the old one, and about whether the new theory really can reproduce all the achievements of the old one. Usually, both theories are about the same phenomena, and the controversies are possible only because, essentially, both the defenders of the old and the new theory use the same language. The languages of the adherents of both theories are the same only 'essentially', not strictly. That is, there are some few - but important - terms in which both languages differ: these terms either are simply different or they are identical but are used in different ways (have different meaning) in both theories.

In addition, such periods may be distinguished from other types of developments by means of psychological and sociological features of the behaviour of the individuals and groups adhering to the two theories, as described in particular by Thomas Kuhn. I will deliberately neglect these features here and concentrate on those conceptual issues which can be stated in an extensional language. It is not entirely clear whether the psycho-sociological features are essential or necessary to the phenomenon of incommensurability: In the absence of any group-fighting would we say that we are confronted with incommensurable theories? Very likely we wouldn't. So the phenomenon would have an essentially intentional character, and my neglecting this would render my whole account inadequate. To this I have two replies. First, we all know that essentiality comes in degrees, and with respect to incommensurability the purely conceptual aspects are much more essential than are the intentional ones. So my account of incommensurability may miss some admittedly important features to be added by psychologists and sociologists. But - secondly - if I were given the choice between really clarifying some phenomenon in *some* essential aspects or between making only some very broad and vague statements in order to be sure not to commit any slight inadequacy, I would tend to prefer the first alternative.

More concrete examples of the kind indicated above are given in the following list which by now may be called standard: Aristotelian theory of motion versus (pre-Cartesian) kinematics; Ptolemean versus Copernican theory of planetary motions; impetus theory versus Newtonian mechanics; phlogiston theory versus stoichiometry; phenomenological thermodynamics versus statistical mechanics; classical versus special relativistic mechanics. If we study these examples on the conceptual level, i.e. if we try to state the respective theories with a sufficient degree of precision and comprehension, we detect the following general pattern. Whenever we try to *match* the concepts of the two theories one by one we will reach a stage in which not all concepts are as yet matched up but in which it is also impossible to continue the process of matching without getting into conflict with what both theories require of their concepts.

More precisely, the situation will be this. We start making a manual for matching each concept of the old theory with a corresponding concept of the new theory,[5] and we succeed in including most of the concepts in this manual. However, at a certain point in this process the following problem arises. The concepts of the old theory, T, are strongly interrelated in T. That is, T contains many assumptions, laws and hypotheses, by which the concepts of T get linked to each other and get determined by each other. A concept of T cannot be explained or learned in isolation but only 'in the context of T', that is, in one process simultaneously with many other concepts of T. This may be expressed by saying that T's concepts *fit into the structure of* T. Now the problem arising in the endeavour to match both theories' concepts is that, at a certain point, we cannot continue matching without destroying the way in which the concepts matched fit into the structures of their respective theories.

In other words, if we succeed in matching the concepts $c_1, ..., c_n$ of T with concepts $c'_1, ..., c'_n$ of the new theory T', such that $c_1, ..., c_n$ fit into the structure of T and $c'_1, ..., c'_n$ fit into the structure of T', it will not be possible to include another concept c_{n+1} of T. That is, for given c_{n+1}, whatever concept c'_{n+1} of T' we choose as a candidate to match with c_{n+1}, the chosen concept c'_{n+1} will not fit into the structure of T'.

[5]The term 'translation' is avoided on purpose. See Section IV.

Expressed still differently: the way both theories' concepts fit into the respective structures does not allow for matching the concepts one by one. (The assumption of a 'one by one' match is not really essential here, and is used only for reasons of simplicity.)

The basic phenomenon of incommensurability at the conceptual level thus consists of a tension between a one by one match of the concepts and the way these fit into the structures of both theories. It is intuitively clear that this tension is a feature entirely internal to the two theories and their conceptual correlation. No intentional aspects are involved and reference to meaning ('meaning variance') and translation is completely avoided. This latter feature deserves special attention, for most of the discussions usually ended up with questions of meaning or meaning variance. In particular, this holds for discussions referring to the notion of translation, for it does not seem possible to say what a translation is without being able to say what the meaning is of some suitable chunks of a language. But there is no good theory of meaning that would help here, and so the people engaged in such discussions usually end up 'with mud on their head', as Paul Feyerabend put it.[6]

Before I proceed to make this basic phenomenon more precise, a picture may serve to further clarify the intuition. Suppose the concepts of our two theories are represented by the nodes in Fig. 1 below, and the way the concepts fit into the structure of T and T', respectively, is represented by the structure as established by the arcs drawn between some nodes. It is clear that no way of matching the concepts (nodes) in the left-hand structure with concepts in the right-hand structure can preserve all the relations between the concepts. If, for instance, we match the concepts as indicated by the dotted arrows in Fig. 1 then c_1, and c_2 will be related in T but their counterparts c_1' and c_2' in T' will not be related (there is no arc between $'c_1$ and c_2' in T'). There is a tension between the one-by-one match of the nodes on both sides, and the ways in which the nodes are interrelated in the two different structures.

[6] In (Feyerabend, 1977), p. 363.

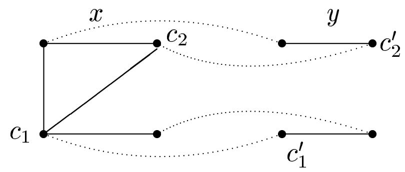

Fig. 1

2. A PRELIMINARY APPROACH

Let us now see how the phenomenon described may be conceptually clarified. Let T and T' be two theories. If t_i denotes an arbitrary word or term from the vocabulary of T then this vocabulary may be written in the form $(t_1, ..., t_i, ...)$. Similarly, we write $(s_1, ..., s_j, ...)$ for the vocabulary of T'. The only assumption about both languages is that they be extensional, otherwise the following definition cannot be stated. Again, it might be objected that this assumption leaves out essential features, and our reply to this is the same as that concerning intentions in the preceding section.

Suppose we have some concrete system x before us, which is an intended application of theory T. Then all of T's terms can be interpreted in x. That is, for each term t_i of T we can point out some entity which is a 'part' of the system x and which in the context of T typically is denoted by t_i. This entity is called the *interpretation* or the *denotation* of t_i in x, and we will denote it by t_i^x in the following. There may be actual problems in precisely determining t_i^x but it is commonly assumed that this entity at least exists, however limited our access to it. If we also accept the (rather trivial) additional assumption that if may be conceptualized as a (possibly very complicated) set then we have come to the point where model theory begins. It is not necessary here to go into the technicalities of model theory. All we need is some agreement that each term t_i of T has an interpretation t_i^x in any concrete system x. If we forget about those features of x which are irrelevant from the point of view of T we may identify the system x with the collection of interpretations of T's terms ('in x'), and write $x = (t_1^x, ..., t_i^x, ...)$ whereby some assignment of interpretations to terms is assumed as given (for instance by the indices: t_i^x is the interpretation of term t_i). Furthermore, it is convenient to consider not only real systems but abstract ones as well. If an abstract entity is given by a collection $(\bar{t}_1, ..., \bar{t}_n, ...)$ of sets such that each set $\bar{t}_i$ has the right type to be an interpretation of the term t_i, we may treat such an entity as a system as well, and extend the notion of an interpretation to such abstract systems. We say that T's terms have interpretations in all abstract systems $x = (\bar{t}_1, ..., \bar{t}_i, ...)$ which are of the right type, and the interpretation t_i^x of t_i in $x = (\bar{t}_1, ..., \bar{t}_i, ...)$ is just $\bar{t}_i$.

Now a *model* of T is a (real or abstract) system $x = (t_1^i, ..., t_n^i, ...)$ such that each t_i^x is an interpretation of term t_i, (in the system), and such that all t_i^x are connected with each other precisely in the way expressed by the statements of T by means of the terms t_i. This is just an intuitive version of the well-known definition from model theory.[7] These notions can of course be applied to T' and we obtain models of T' in the form $y = (s_1^y, ..., s_j^y, ...)$.

A crucial term in Section 1 was that of a match between the terms of two theories. This notion becomes accessible if we transfer it to the level of models. If $x = (t_1^i, ..., t_n^i, ...)$ and $y = (s_1^y, ..., s_j^y, ...)$ are models of T and T', respectively, we say that x and y *match up* iff for all t_i^x there is precisely one s_j^y such that t_i^x matches with s_j^y and conversely, or equivalently, if there is some permutation π of the indices such that for all i: t_i^x matches with $s_{\pi(i)}^y$. This is of course a purely formal move as long as the meaning of 'match of interpretations' is not explained (more on this below). For the moment let's stick with the vague expression which seems to be quite intuitive in concrete cases. If t_i^x, for instance, is the mass function in a model x of classical mechanics, and if s_j^y is the mass function in a model y of special relativistic mechanics such that both x and y 'describe' the same, real, moving particles it is clear that t_i^x matches with s_j^y. If s_j^y, on the other hand, is the global force function of the relativistic model then, clearly, t_i^x does not not match with s_j^y.

In order to state our definition we have to extend the notion of 'match' to cover cases of 'partial match', too. If S is a subset of the vocabulary of T we say that models x of T and y of T' *match in* S iff, for all w in S, the interpretation w^x of w in x matches with some member w^y of y and conversely. If, for instance, S contains only kinematical terms, and x, y are, as before, classical and relativistic models of 'the same system'

[7] See, for instance, Shoenfield (1964).

then x and y intuitively match in S : there is a natural correspondence between, say, the particles, the instants, and the position functions in both models, namely identity. If, on the other hand, y is a model of the motion of quite different particles there will be doubts as to whether x and y match in S.

Now we can state a preliminary definition of incommensurability which will be emended further below. Suppose we have two theories T and T' before us such that our notational conventions apply. Both T and T' are about the same phenomena, so their joint vocabulary, i.e. those terms common to T and to T', will be rather large relative to all the terms of T and T'. In particular, the joint vocabulary will be non-empty. Let x and y be given models of T and T', respectively. (It will be helpful to think of x and y as 'describing' the 'same' real system from different points of view, maybe by using different terms as provided by T and T'). The idea developed in Section 1 was to start matching the terms of both theories and see where this clashes with how they fit into the structures of both theories.

Quite naturally, one will start matching terms from the joint vocabulary first, among those one will concentrate on the unproblematic ones, i.e. on those for which there is some straightforward correspondence like identity or identity of meaning. Suppose in this way we succeeded in matching the terms of a relatively large subset S of the joint vocabulary. But if S is sufficiently large ('maximal' in this respect) we cannot extend the match to further terms without running into problems, that is, without contradicting the basic laws of both theories. In terms of models this means that we cannot extend the match between two structures x, y beyond S and still insist that both x and y are models (of T and T', respectively). In particular, the match cannot be extended to the *full* joint vocabulary. In the following we will concentrate on the latter special case, first, because it facilitates the formulation, and second, because it still covers the intended examples. In summary, we obtain the following simple condition. For some suitable subset S of the joint vocabulary of T and T', and for any two models x of T and y of T': if x and y match in S then it is impossible to extend this match to the full joint vocabulary of T and T'. An equivalent formulation stressing the clash with the assumptions of both theories is this. There is no suitable subset S of the joint vocabulary such that for all structures x, y: if x and y match in S then it cannot be the case that x is a model of T, y is a model of T' and x and y match in the full joint vocabulary. By adding the requirement of the joint vocabulary being non-empty we obtain the following preliminary characterization of incommensurability$_p$ (with 'p' for 'preliminary').

Theories T and T' are *incommensurable*$_p$ iff
(1) the joint vocabulary of T and T' is non-empty
(2) there is some non-empty subset S of the joint vocabulary such that for all x, y: if x is a model of T and y is a model of T' and x and y match in S then x and y do not match in the full joint vocabulary.

Note that condition (1) follows from (2). This account immediately raises the following question: which subset S of comparable terms should be chosen to do the job? Thomas Kuhn in a private discussion at once pointed out that it is not easy to justify some particular choice of terms as those for which comparison ('match') is unproblematic. For this amounts to drawing a distinction among the terms of a theory such that the meaning of one subclass of terms is independent of the meaning of the other subclass. A holistic picture about science and about meanings of terms in a theory throws some doubt on the possibility of such a distinction. In addition, if we had criteria for the choice of the 'right' class of comparable terms we would also have a criteria for distinguishing some sound empirical or observational basis of theories which is not affected by 'turbulences' on the theoretical 'Überbau' - and thus criteria for continuity and progress. Of course, this does not completely demolish the idea of choosing some correct class of comparable terms, for we are not without any clue to 'the right' choice. It may be pointed out that terms which have 'lived through' a long scientific development without major modifications (like Euclidean distance or mass (= inertial mass = rest mass)) have achieved some dignity which gives them enough independence to serve as a basis of comparison. Also, it may be pointed out that terms get more standing in connection with their multiple referents, that is, with multiple established and important links from other theories (like 'mass' in mechanics which is linked to the weight-function in stoichiometry, to energy in thermodynamics, to the stress-energy tensor in general relativity and so on). These possibilities notwithstanding it seems that we are far away from having interesting and applicable criteria in order to choose 'the correct' set of comparable, common terms (if there is such a thing).

There is a simple way out of this problem: just strengthen the above condition (2) to hold for *all* proper subsets of the joint vocabulary. So T and T' would be incommensurable$_p$ iff condition (2) above is replaced by

($2'$) for all proper subsets S of the joint vocabulary and for all x, y:
if x is a model of T and y is a model of T' and x and y match in S then x and y do not match in all of the joint vocabulary.

Though such a condition may be cumbersome to check in concrete cases it seems a move in the right direction. If commensurability has holistic roots insofar as it does not allow one to break both theories' structures apart for the sake of one by one comparison then there is no reason why such one by one comparison should be justified for just one distinguished subclass of terms. Rather what should be expected is a kind of symmetry: whatever subclass we take as a basis of comparison we will end up in difficulties. So let's accept this move as an emendation. The transition from (2) to ($2'$) only apparently complicates the issue. On closer inspection it turns out to simplify things considerably. For now reference to a subset S of common terms in the 'if'-clause of condition ($2'$) becomes redundant. In fact, ($2'$) is obviously equivalent to the following

(2^*) for all x, y : if x is a model of T and y a model of T' then x and y do not match in all of the joint vocabulary.

3. REQUIREMENTS FOR COMMENSURABILITY

Still there remains another problem, namely to make precise the meaning of 'to match', and by 'solving' this problem we arrive at the desired characterization of incommensurability. The problem may be stated as a problem of choosing the correct solution from a whole spectrum of possible solutions. This spectrum ranges from taking 'to match' simply as 'to be identical' on the one extreme to defining 'to match' by means of some specific set theoretic function or relation at the other extreme. If we take 'to match' as 'to be identical' then two models x, y match in S (where S is a subset of the joint vocabulary) iff for any term t in S its interpretations t^x in x and t^y in y are identical. Since we agreed on the set theoretic nature of all interpretations this amounts to stating that the two sets t^x and t^y - however complicated - are identical, and set theory provides a clear and easy criterion here. On the other hand the weakest definition of 'match' would be to require that the interpretations of a term in two models of both theories be merely related by some set theoretic relation. The term 'state', for instance, in a model of thermodynamics is interpreted by unspecified, basic entities whereas in a model of statistical mechanics it is interpreted by sequences of function values for positions and momenta. The classes of these different entities may well be related with each other by some set theoretic relation.

In between these two extremes there are many other possibilities, like replacing set theoretic relations by relations definable in various different ways in formal calculi of different logical properties and strength.[8] Intuitively, the closer we get to 'identity' as defining 'match' the weaker our concept of incommensurability. For the preliminary definition says that a match in a given subset S can not be extended to a full match. And the more we require for 'match' the less we have to show in a proof that terms can not be matched. On the other hand, the weaker we choose our definition of 'match' the stronger the resulting concept of incommensurability.

A brief reflection shows that the extremely weak definition of 'match' as a mere set theoretic relation yields a concept of incommensurability so strong that it hardly will have any real instances in the history of science. For in this case we usually will be able to establish set theoretic relations - contrived ones or plausible ones - among the interpretations of all the common terms. (Roughly, between two classes of entities there always exist some set theoretic relation.) On these grounds we may dismiss this first possibility.

Next, as concerns the intermediate cases, it seems that to adopt one of these will make the notion of incommensurability dependent on a particular logical system, its strength, or on some special syntactic features of definitions. This also is reason enough to dismiss such possibilities for it would be strange that an important meta-scientific and philosophical concept should depend on the logical subtleties mentioned. I admit that this is a rash conclusion, and I will remain open for intermediate cases

[8] One such intermediate possibility is found in Graham Oddie's contribution to this volume. Alas, its effect is to turn all the examples from Sec. 1 into commensurable ones. Another possibility is based on a frame used by David Pearce. See, for instance, (Pearce, 1982) and (Balzer, 1985a, 1985b).

which do not depend on such subtleties. For the time being, however, and in the absence of 'interesting' intermediate proposals (which also have to save the phenomena) I think that the remaining extreme case is the most plausible and adequate one. It is plausible because it is simple and easily applicable; it is adequate because it mirrors actual talk in scientific dispute and because it can successfully deal with the examples mentioned in Section 1 (see also Section 5 below) while not turning other real-life examples of non-incommensurable theories into incommensurable ones. I therefore adopt 'identity' as the correct solution for the definition of 'match'. Consequently I will say that, for some subset S of the joint vocabulary of T and T', and for two models x of T and y of T', x and y *match in S in identity* iff for all terms in S the interpretations t^x and t^y of t in x and y are identical. We than have the following final requirement for incommensurability.

Theories T and T' are *incommensurable* iff
(1) the joint vocabulary of T and T' is non-empty
(2) for all x, y: if x is a model of T and y is a model of T' then x and y do not match in the joint vocabulary in identity.

Some remarks may be added. First, it has to be stressed that condition (1) is only a rather poor expression of what we intend to cover, namely that the joint vocabulary of both theories is rather large in comparison with the union of both vocabularies. In general, it would be too much to require that both theories have the same vocabulary. But as soon as we admit differences it becomes very difficult to say that what subset of joint terms is 'relatively large'. In the absence of any reasonable criterion we retreat to the much weaker condition (1) on the purely formal side. In real-life examples the worst deviation from full identity of the languages will be cases where one theory (or both) contains very few (one, two or three) 'theoretical' concepts not available in the other theory.

Second, it has to be stressed that these requirements only provide a basic core for a theory of incommensurability. Various specializations due to the particular circumstances in particular real examples are to be expected (see Section 5).

Third, the logician will be eager to point out that these requirements are equivalent to the definition of two theories' being inconsistent. I agree.

4. POSSIBLE OBJECTIONS

A first objection to this approach is that it does not refer to meaning and translation. Usually, the subject is discussed in terms of meaning variance or untranslatability. As already mentioned I have some reservations about such discussions. Not that I want to say they are useless. But it seems to me that the phenomena at hand can be conceptualized and clarified in less ambitious terminology. To refer to translation and meaning is to rely on deep philosophical issues which are far from being clearly understood up to now in order to deal with phenomena that do not reach quite as deep into overall philosophical themes. (Still, incommensurability is interesting enough and certainly not a surface-phenomenon.)

There was some discussion with David Pearce[9] recently who suggested a two level picture with the level of meaning represented by some kind of set theoretic relation among models (a 'reduction relation') and the level of language represented by a 'translation', i.e. a mapping of the sentences of both languages satisfying certain additional properties. It was clear from the beginning that a set theoretic relation among models does not cover all aspects of meaning relevant for the comparison of theories. What became clear in the course of the discussion is, I believe, that the same holds for translation. We are far away from being able to compress all the aspects of the notion of translation into a mapping of the sentences satisfying certain additional, precise requirements. These brief considerations confirm what was said in the last paragraph. To summarize the point: it seems to me that the absence of meaning and translation does not raise any objection against my account. On the contrary, it has to be counted as one of its positive features.

A second objection focusses on my central use of identities: considerable identity of the vocabularies, and identities of the interpretations of the common terms. From a logician's point of view it may be said that identity of the vocabulary cannot be too important because of the possibility of replacing ('renaming') the established terms by new, say, artificial ones without changing the theory. Identity of interpretation may be found unsatisfactory because in first-order logic predicates can only be characterized up to isomorphism. The first part of the objection concerning identity of terms in fact shows that the present definition is

[9] See the references in note 8 and also (Pearce, 1986).

still too limited with respect to the possibility of 'equivalent' reformulations of theories. If T and T' are incommensurable in the sense above it still may be the case that we find *real-life* reformulations T_1 of T and T_1' of T' such that T_1 and T_1' are no longer incommensurable (because, for instance, they have no terms at all in common). To this objection two things can be said. First, we may extend the definition to cover reformulations in the following way. We say that T and T' are *invariantly incommensurable* iff there exist equivalent versions T_1 of T and T_1' of T' such that T_1 and T_1' are incommensurable in the sense defined above. The second observation to the point is critical with respect to the actual importance of such reformulations. Of course, we know the standard examples of supposedly equivalent formulations, like Newtonian and Lagrangian mechanics, or matrix- and wave-formulations of early quantum mechanics. We do not know, however, of any detailed analysis of such examples that demonstrates equivalence, nor do we know general concepts of equivalence that were successful when applied to concrete examples of the kind mentioned. Our own recent attempts[10] tend to support this negative picture. So there is some doubt, to put it mildly, as to whether the idea of equivalent formulations is an important one as far as real cases are concerned.

The second part of the objection, namely the one concerned with identity of interpretations, from my point of view only demonstrates once more the limitations of first-order logic, and I would hesitate to accept these restrictions just on the basis of the 'beauty' of completeness theorems, and compactness and Loewenheim-Skolem properties. Moreover, it seems possible to provide a formal treatment of my account that takes interpretations as basic objects (of a many-sorted first-order language), and within such a treatment the identities under discussion may be stated without any problems.

A third objection is that my characterization includes inconsistent pairs of theories, or even more sharply, that it consists simply of the requirements for inconsistency. The natural way to define inconsistency of two theories is by requiring that they have no joint model, that is, no structure for the union of all terms which satisfies the axioms of both theories. This is, in fact, equivalent to the above requirements. But, so the objection, inconsistency is a case of commensurability because

[10] See (Balzer, Moulines, Sneed, 1987), Chap. VI.

inconsistent theories can easily be compared (the negative outcome of the comparison does not count against its being some comparison). Again, there are two points in reply. First, I have some difficulties in relating the terms 'incommensurability', 'commensurability' and 'comparability' with each other. Even the relation between the first two is not one of simple negation because this would make theories commensurable which have no terms in common at all. It seems that condition (1) above should apply also in cases of commensurability, so that the relation is this:

incommensurability = condition (1) and condition (2)
commensurability = condition (1) and condition (not-2).

With respect to comparability I believe that the central adherents of incommensurability - Feyerabend and Kuhn - did not claim that incommensurability implies incomparability. On the contrary. Incommensurable theories can be and are compared with each other - even if such comparison usually takes place much later than the original quarrels. It may well be that at the moment we are not very smart in comparing incommensurable theories. But structuralist, holistic accounts of comparison (like 'reduction' relations) seem to be possible even in the light of incommensurability. Therefore I would like to deny that comparability implies commensurability. But then incommensurability does not imply incomparability, and the objection fails.

Secondly, as already mentioned, my requirements 'boil down' to those of 'mere' inconsistency. As Feyerabend held a similar view long ago, what's new? I think the value of the present paper is not in its result (= requirements for incommensurability) but rather in the 'derivation' of the result. The way in which the result was reached is independent of the writings and explications of other authors. If, in such a situation, the same result occurs, the better for the result. It is confirmed (according to the bootstrap view of confirmation). I take this as an essential contribution of this paper: to confirm that incommensurability is just inconsistency.

A last objection is that my account is not operational. In order to check whether two theories are incommensurable we have to find out identities about their respective interpretations. That is, we have to find out whether two entities (objects, relations, functions) denoted by the same term in two models of the respective theories are identical. This affords criteria for identity independent of both theories, and thus a kind of 'Archimedian point' for the comparison. But such an independent point of view is feasible only for the metaphysical realist (it is given by 'reality'). Therefore, so the objection, this account leads to metaphysical realism. The short reply which is dictated here by reasons of space (and which perhaps will not be compelling for many readers) refers to the distinction between form and content. There are certain things and relations which we have learned to accept by purely formal means; from this fact logics derives its right to exist. The comparison of two given, axiomatized theories according to my definition may be regarded as a purely formal set theoretic exercise, and the claim associated with my definition is that this exercise in the real-life examples mentioned above (once they are axiomatized) yields positive (i.e. incommensurable) examples.

Still, one may feel uncomfortable with this and wonder how the realist's problem of checking 'cross-theory' or 'cross-world' identities is circumvented or neutralized by purely formal means. Roughly, this happens as follows. From a realist, non-formal perspective, condition (2) which involves the identities under discussion may be checked for 'real' cases, that is, interpretations occurring in 'real' systems. T and T' would be incommensurable if no match is possible in this domain of real systems. But a formal view includes many more, 'abstract' systems (models) as well. A priori there might be cases where there is no match in the domain of real systems but where there exist two abstract models that can be matched. Such a case would come out as non-incommensurable from the formal point of view adopted here, but it might be claimed to constitute an example of incommensurability by the realist. This shows that the formal perspective yields a much stronger concept of incommensurability. In view of the real cases to be covered it simply happens that this more narrow concept nicely applies. I do not want to belittle the problem of cross-world identities, I just want to say that a theory of incommensurability can go along without paying too much attention to this problem.

5. A BRIEF LOOK AT EXAMPLES

To conclude, let us really look at the phenomena as given by the examples mentioned in Section 1, and see how the 'new theory' applies to these cases. Each example on its own certainly needs an extensive treatment: first reconstruct the two theories involved, and then check whether the above requirements apply. It is clear that this is a program for a whole book. So I have to be very brief and sketchy (as everybody is in this context when it comes to examples).

The vocabularies of Aristotelian theory of motion and pre-cartesian kinematics do not have special terms associated with them. So both vocabularies by and large can be taken to be identical. The crucial tension may be located around the term 'motion' which in Aristotle is much more comprehensive, including changes of features different from position. If we identify the interpretations in two models of the other terms strongly linked with 'motion' we see that the interpretation of 'motion' in an Aristotelian model is different from that in a kinematical model.[11]

Similarly, the vocabularies of the Ptolemean and Copernican theories of planetary motion are essentially the same. Apart from ordinary language the technical terms also seem to coincide.[12] But clearly there are terms the interpretations of which in the (unique) two models are different, for instance 'centre of the sun's path' which is the centre of the earth in Ptolemy's and a point inside the sun in Copernicus' theory.

In phlogiston theory there is the term 'phlogiston' in addition to Lavoisier's terminology, and conversely, 'oxygen' does not occur in phlogiston theory. As Thomas Kuhn argued in his PSA paper there is a group of terms to which condition (2) above nicely applies,[13] and he convincingly worked out the tension between the theoretical structures on both sides and between one by one identification of those terms.

In impetus theory and Newtonian mechanics the vocabularies are roughly the same: the technical terms are present on both sides. A term creating problems of comparison is, for example, the term 'natural motion'. In models of impetus theory this term denotes the path of a particle at rest relative to the surface of the earth. In Newtonian mechanics it denotes the path of a particle at rest or in uniform motion relative to an 'inertial system'. Since the surface of the earth is not an inertial system the term has different interpretations in any two models of the respective theories. It has to be noted that this informal sketch leaves implicit a crucial point, namely the characterization of inertial systems in Newton's theory. In my opinion, if we reject recourse to metaphysics, inertial sys-

[11] See (Kuhn, 1981) from which the example is drawn, for further details.
[12] Compare (Heidelberger, 1976).
[13] See (Kuhn, 1983).

tems can only be characterized by second-order sentences talking about sets of models of the theory. If this is so then we have here a first example which for its proper treatment requires some specialization of the general core. The 'specialization' (on the meta-level of incommensurability) here will have to include the full range of specializations ('special laws' at the level of mechanics) of both theories involved, and therefore will yield a notion of incommensurability for whole theory-nets.[14] This is, of course, only a hint and will have to be worked out in detail.

The last example I want to consider is that of classical and (special) relativistic mechanics.[15] Here, the only difference in the vocabularies is that 'velocity of light' in the relativistic theory has acquired the status of a technical term. At a first intuitive glance it seems that condition (2) fails because in models with zero-forces the masses on both sides will coincide, and therefore there are joint models of both theories.[16] There is, however, a little problem with this intuitive account for it neglects that, properly speaking, the term 'mass' is not the same in both theories. Sure, we use the same word in ordinary talk but we also admit from the beginning that mass in the relativistic theory depends on velocity whereas it does not depend on velocity in the classical theory. So, properly speaking, the term has different type in both theories and it therefore would be dumb to try checking whether it has identical interpretations in two models: it cannot have, for reasons of typification. If this is accepted, another problem arises. If 'mass' is not a common term then it need not be considered in the evaluation of condition (2). But then the two theories become 'commensurable' (more precisely: non-incommensurable), for the other terms maybe interpreted identically on both sides at least in some models. Here we have another case which requires specialization of the basic picture. Intuitively, the adequate treatment will be to include the term 'mass' in the attempt of matching but to relax the condition of identity for its interpretations. Formally, the corresponding specialization may be stated by extending condition (2) to certain well specified

[14] See (Balzer, Moulines, Sneed,1987), Chap. VI.

[15] I have nothing to say about thermodynamics and statistical mechanics, mainly because of the lack of attempts at working out the structure of statistical mechanics. Obviously, mere identity of interpretations of common terms (like 'state') will not do. Some other specialization - typical for 'micro-reductions' in general, perhaps - will be needed.

[16] I here refer to the reconstructions provided by (McKinsey et al., 1953) and (Rubin & Suppes, 1954).

cases of terms which are not common to both theories but which have some strong, say, syntactic similarity (to be precisely specified).

Still the case is not settled, it really is a borderline case. Things will now depend on precisely how we relax the identity requirement for interpretations. The most natural way for the present case (which may be representative for many similar cases in which just a new argument of a well-known function is 'discovered', and the theory adjusted accordingly) is to replace identity by some relation of 'being part of'. The classical mass function in the crucial case of zero-forces is just a 'part of' the relativistic mass function, namely in the sense that each pair (particle p, mass of p) is a component of the corresponding triple: (particle p, velocity of p, velocity dependent mass of p). Thus condition (2) becomes: for any two models x, y it is not the case that (all their interpretations in the joint vocabulary are identical and their interpretations of 'similar' terms are related by the 'part of' relation). In the example before us this condition fails because there are models in which the classical mass function, in fact, is contained in the relativistic one in the sense just defined. The conclusion then is that classical and special relativistic mechanics are commensurable. This result is in accordance with judgements of physicists which have taken seriously the idea of incommensurability (the majority, for which this qualification does not apply, will of course simply deny any phenomena of incommensurability in physics).

It has to be stressed that this result depends on a particular view of special relativistic mechanics,[17] namely as 'being built' on individual frames of reference which have a classical space-time structure. I used this approach here for reasons of simplicity, but I do not at all want to defend it. I would conjecture that the inclusion of the level of space-time (at which the problem of comparison properly has to be considered) in the reconstruction of the dynamical theories will yield a case of incommensurability,[18] just as will the study of the relation of classical and general relativistic space-time and mechanics.[19]

[17] I here refer to the reconstructions provided by (McKinsey et al., 1953) and (Rubin & Suppes, 1954).

[18] See (Balzer, 1984) for an axiomatic attempt at comparison at the level of space-time.

[19] See, however, (Ehlers, 1986) and (Malament,1979) for different opinions.

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