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The Triplet Structure of Concepts

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Summary

We introduce a precise model for the theory of concepts in philosophy of science. In this model we connect the level of description, the level of reality and the level of set theoretic systems. On the one hand we describe a general frame for the collection of concepts, and, on the other hand, the 'local' structure of a concept. We specialize this frame to scientific concepts, scientific theories, and to the appertaining structuralist constructions from theory of science.

Zusammenfassung

Wir stellen ein präzises Modell der wissenschaftlichen Begriffstheorie vor, in dem die Beschreibungs-, die Wirklichkeits- und die mengentheoretische Ebene verknüpft werden. Einerseits wird ein allgemeiner Rahmen für die Gesamtheit der Begriffe, andererseits die 'lokale' Struktur eines Begriffs beschrieben. Wir spezialisieren diesen Rahmen auf wissenschaftliche Begriffe, wissenschaftliche Theorien, und auf die zugehörigen strukturalistischen, wissenschaftstheoretischen Konstruktionen.

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1 Introduction

We specialize a new approach, the *triplet approach to cocnept analysis*, which has been published by Kuznetsov in (1994, 1996a, b, 1997, 1998, 1999, 2002, 2003, 2004) and refine two specific points.

Before we turn to our actual goal, we would like to localize our work within the dynamic and diverse network of scientific approaches. Thorough descriptions of the many different aspects, approaches, 'theories', reasons and goals can be found in Weitz (1988), Ros (1989, 1990a, b) and Wachman (2000), for example. Some other aspects were taken mainly from Internet sources. In a passage from Hjorland, (2009 p. 9), four important viewpoints were concisely stated: in empiricism, concepts are mainly classifications of similar 'objects', in rationalism they are 'constructed' through given, formal means of definition. In the historical-dynamic approach, concepts are generated through genealogy and through relationships with theories and discourses, and in pragmatics, one decides whether a concept is best suited for a certain class of 'things' for a certain purpose, and also, if a symbol should be used for this concept. In a fifth materialistic-dialectical perspective, a material basis is presupposed from which the society and the appertaining concepts can unfold and become more complex and reach a higher level of quality. These five perspectives are not disjoint but are often formulated in a very narrow and aggressive manner.

The triplet approach uses all five perspectives. It contains a component which names the real entities, thereby accommodating the empirical aspect. It also has a rational portion containing means of definition and construction, used in set theory. The historical-dynamic developmental aspects of a concept, which are available at a linguistic level and in a knowledge system of a concept, was described in Kuznetsov (2004). We would not go in to further detail on this point as we have chosen to limit the theme to the static part of the model. This applies to the pragmatic[3] and materialistic-dialectical[4] approaches as well.

Many of the explicit and implicit parts found in our triplet model have been used in earlier approaches. The classifying aspect (Sec. 5) has been known since antiquity. Extent, content, extension and intension were topics of fierce debate in medieval times, see e.g. Weitz (1988), Chap. 3. These aspects too, are found in the triplet model. Our construct of a concept corresponds, in part, to the content, whereas the three other components are only implicitly available in our model (Secs. 2 and 4). The difference between meaning and sense, which Frege discussed in 1892, could be copied into the triplet model (Sec. 4). In logical empiricism, this difference was expanded using formal means ('possible worlds').[5] The materialistic-dialectical stream, discussed in the last

[3] See (Burgin and Kuznetsov 1994).

[4] (Kuznetsov 1997).

[5] This aspect cannot be discussed further at this time.

century mainly in the Soviet Union and socialist countries[6] flowed of course into the triplet model. The historical-dynamic component, especially the one cited by Kuhn[7] cannot be a natural part of our structural model. In contrast, in the triplet model, as in many earlier works,[8] *meaning* (in today's dominant language: *reference*) plays a pivotal role. Especially, in approaches from theory of science two distinctions for concepts introduced by Carnap (1966) and Sneed (1971) must be mentioned, which too, belong to the scope of our model (Sec. 4).

In the help of these rough sketches we claim that, on the one hand, the triplet approach encompasses all important points of other approaches to concept research. Some earlier approaches can be identified with substructures of our model; others can be understood in our model as limiting cases. On the other hand, our model introduces two new aspects which are either not to be found in other approaches or merely touched upon in a rudimentary way. First, we specify the relationship between the inner structure of a concept and the knowledge system. Secondly, we can investigate many previously unstudied relationships between a concept and the changes of this concept with very little expense.

The goal of our work is, on the one hand, to formalize and specialize the triplet-approach, so that it can be applied to formal accounts – especially from theory of science – as well as to other scientific disciplines. The general triplet model uses a *knowledge system* as a basic building block for concept analysis, for example Kuznetsov (1997). Here we limit ourselves to knowledge systems described in the set theory of Bourbaki and use the notation of the structuralistic theory of science.[9]

On the other hand, we would like to take back the role of natural language/languages a bit, which in most approaches takes a dominant role by discussions of concepts. Of course, also our approach must be formulated in a natural language. We think, however, that our representation may be expressed in every other natural language embodied in a set theory. This goes to say that we take pains to promote as linguistically neutral a description as possible.

In the latest discussions about concepts, a new psychological aspect of individualization plays a role, namely the *possession* of concepts, see e.g. Peacock (1992). This aspect, from a linguistic point of view, was criticized by Fodor (1998) and further discussed, for example, in Prinz (2002), Murphy (2002) or Gleitman and Gleitman (2007). Our model gets along without the psychological level of attitudes and beliefs of single persons. The structural notation we use may be found in a similar form in Diez (2002), however here on the psychological level.

Finally, we must mention that our account uses several aspects of idealization. We idealize in order to avoid describing all the natural ramifications.[10] In return for

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[6]See (Gorskiy, 1983).

[7]See (Kuhn 1974) and later (Andersen et al. 2006). These components are now also studied with formal means (Cohen and Murphy 1984), (Gabora and Aerts 2002).

[8]For instance in (Putnam 1975).

[9](Bourbaki 2004), (Balzer et al. 1987), (Diederich et al. 1989, 1994).

[10]Several come up in the second example at the end of Sec. 3.

these idealizations we can describe the basic structure of concepts in a rather clear and simple way.

2 A Frame for Concepts

We introduce a frame R in which all concepts, the common as well as the scientific, can be at home. This frame contains nine components:

$$\langle Ph, W, M, des, int, \varphi, B, \diamond, \Sigma \rangle.$$

In the triplet approach, the three ensembles Ph, W, M form a basis from which all parts of a concept are built.

Ph is the ensemble of all designators of concepts which exist in different languages. An element from this ensemble we call a 'description for something that belongs to a concept' or simply a 'description of a concept'. We claim that there are no full descriptions for any concept. We idealize this ensemble into a blurred or fuzzy set of precise and imprecise descriptions of concepts that we will denote in the following by *phrases* – in the linguistic sense.[11] In first approximation a phrase is a list of words which together express a meaning. In borderline cases, a phrase is a list containing just one word or symbol. In linguistics, phrases are categorized in nominal phrases, verbal phrases, and other phrases. In the following the phrases are printed in fat. For example, nominal phrases are **tree**; **ein Baum**; **the trees**, or more complicated **Peter, the man on the moon**; **the rockets which were shot off during the last war**. In Indo-Germanic languages, the verbal phrases, which can be simple or complex, are also important: for example, **to go**; **going to a restaurant with a friend**; **to win a battle**. Each phrase belongs to a particular natural language or to several even.

W is the ensemble containing the *real entities*. Each single real entity can be directly experienced only with the help of the human senses. All other information about a real entity we receive only from linguistic detours. The set of *all* real entities has a quasi transcendental character.

The ensemble M makes up, in general, the level of knowledge systems. These systems are so manifold, that we simplify and idealize them to a set of *set-theoretic constructs*.[12] In set theory a construct *is* first of all a set, even if can have a complex, inner structure. We emphasize this to prevent a naive access to set theory. Set theory is not made up only of processes, where one starts with 'given' objects and then from such objects creates new sets. One can also analyze a set from within without constructing this set from given objects.[13] An element of M can also contain many other components and complex parts. In spite of this, the elements can be converted into a set-theoretical whole – a construct. Depending on the concept, a construct can also be quite simple.

The expressions for sets and the appertaining relations, like for example the relation of being an element of a set ($\in$), the relation of equality ($=$), the subset relation

[11](Bünting 1984), 4.5, (Radford 1981), Chap 3.

[12]In the following we omit the adjective *set-theoretic* and write just *construct*.

[13]See, in contrast, the constructivist approach of Lorenzen (1987).

($\subseteq$), can be normed in different languages in the same way. In our approach, we outsource in a certain way these central set-theoretic relations, to declare them as being generally valid. In other words, we do not mention the set-theoretical basic concepts which appear within our frame. Also in the natural languages, the set-theoretic elements normally remain implicit.

These three ensembles make up three levels which cannot be further reduced: the level of designation, the level of reality and the level of constructs. Correspondingly, every concept contains three basic parts.

The three levels are bound together through several relationships. First, we use a *designator function des.* When b is a description – a phrase – and e an entity, the designator function *des* expresses that b uniquely designates the entity e; for short: $des(b) = e$ and $des : Ph \rightarrow W$. In other words, b is a symbol for something real (e), which in the simplest case can be at this moment seen (smelled, etc.). The claim that each function value of *des* is uniquely determined contains an idealization. Of course there are phrases, which designate several different things. Names are in this sense especially extreme. The hypothesis of uniqueness can be reversed without a problem, however. In our model we formulate the boundaries of reality in very general terms. For instances, the ideas and fantasies of real persons, and the entities which 'live' in computers are also real.

Secondly, we use an *interpretation function int.* To each phrase a set-theoretic construct is assigned, $int : Ph \rightarrow M$. $int(b) = m$ says that the phrase b points uniquely to the interpretation m. This function appears in many different variations under different names. Our term 'interpretation' corresponds to the German term '(Be)Deutung' used in the original paper. Instead of 'interpretation' one could also use the term 'reference'. An interpretation can have aspects, which are directly related to certain persons, whereas the reference is, for the most part, independent of persons. In logics, the term *interpretation* is used in a similar way. A symbol, an utterance, a name, a predicate, or a phrase denotes an object, a thing, a circumstance or a set. Here we use a more general formulation saying that by a phrase 'something' is interpreted 'as a set'. By strong idealization we say that a phrase denotes exactly one interpretation. This requirement can be reversed as well using additional model components. We would like to emphasize that at the level of reality, further distinctions cannot be made. All distinctions have to take place on the levels of phrases or constructs.

We introduce a *relation of correspondence* φ, which has a more theoretical character. A real entity w corresponds to the construct m: $\varphi(w, m)$, thus φ has the form $\varphi \subseteq W \times M$. It is possible to view this relation from both sides. On one side, we can start with a construct (a set) for which we find a corresponding real entity. Reversed, we can start with a real entity, which we perceive, or imagine, and we find a construct – we *construct*. There are constructs (sets) which are so far from reality, that we cannot find any corresponding entities. Also, the reversed situation can be discussed: for an entity we cannot find a corresponding set. This would mean that the universe of sets does not contain means for expressing everything which the human spirit can imagine. The relation of correspondence therefore is not able to form a relationship between *every* set and entity.

Aside from the ensemble Ph of phrases we need to use an subensemble B of *meaningful* phrases and a peculiar entity $\diamond$ which we have settled into the level of reality and refer to as the *meaningless entity*. These two components are necessary in effectively handling the meaningless phrases, which occur in every natural language.

Finally, the frame for concepts contains an ensemble Σ of *natural languages*.[14] In order to simplify, we identify in this article a natural language $S \in \Sigma$ with the set of nominal and verbal phrases of the language, and we abstract all further components of S. For the same reasons we subsume the hybrid elements in jargon and other technical language under natural languages.

For these components we formulate some more obvious assumptions and two hypotheses with important contents.

Let us assume that no meaningful phrase designates the meaningless entity. In other words, all meaningful phrases $b \in B$ designate entities e, which are different from the meaningless entities ($\diamond \notin des(B)$). The ensemble Σ contains finitely many languages $S_1......S_r$. Each language S_i is made up of a set of phrases ($S_i \subseteq Ph$), and all phrases shall be found in the languages ($Ph \subseteq S_1 \cup ... \cup S_r$).

Our first hypothesis (R1) states that by the correspondence relation φ a real entity is related one-to-one (bijectively) to a construct. In other words an entity w which is designated by a phrase b, clearly corresponds to a construct m which is interpretated by the same phrase b:

$$\varphi(des(b)) = \varphi(w) = m = int(b).$$

One could also say that the meaningful entities, named by nominal and verbal phrases are one-to-one assigned to constructs.

The second hypothesis (R2) concerns several languages. When two phrases, b, b' designate the same entity w, $des(b) = w = des(b')$, when the phrase b can be found in two languages S and S', and when b' belongs to language S', then phrase b' is also a part of the language S. Thus when phrase b can be found in both languages then phrase b' can also be found on both languages.

The ensemble of phrases can be easily divided into equivalence classes by the designator function des. Two phrases are equivalent if and only if they designate the same entity. As is normally the case, we can limit such classes to a genuine language.

The function values $des(b)$, which stem from meaningful phrases $b \in B$, form the set $des(B)$. The function values $des(b)$ we call 'meaningful entities'. We are interested here only in the subensemble $des(B)$ of the meaningful entities of W. In the same way we can form the subensemble $int(B)$ of M. $int(B)$ is made up of exactly those constructs which are interpreted by meaningful phrases. Set-theoretically we can restrict the theoretical correspondence relation $\varphi \subseteq W \times M$ to the product $des(B) \times int(B)$, $des(B) \times int(B) \subseteq W \times M$. We require that the correspondence

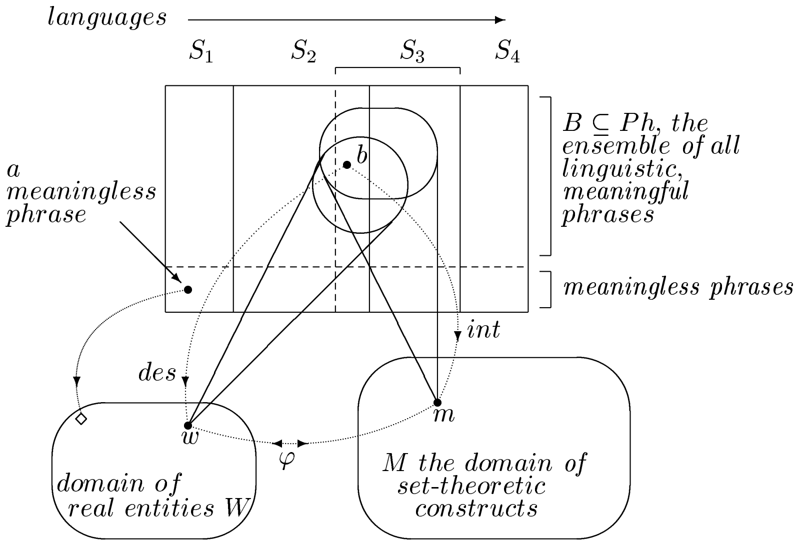
[14] We are choosing not to get involved with formal and virtual languages here.

relation applies only to pairs $\langle e, m\rangle$ of meaningful entities and constructs interpreted by phrases; $\varphi \subseteq des(B) \times int(B)$. Together with hypothesis R1 φ becomes therefore a bijective function $\varphi : des(B) \to int(B)$.

We are well aware that hypothesis R1 leads to metaphysical questions which we cannot and/or do not want to answer. For the correspondence relation, one could probably find counter examples in the direction from constructs to entities. But as we concentrate on a rough structure, such counter examples do not hold much sway. Hypothesis R2 could also be refuted by examples. We find this idealized assumption purposeful even in those limiting instances, where it is not valid. This hypothesis says, in other words, that phrases in different languages, which overlap, can be collected into equivalence classes through *des* and through *int*.

Many aspects of this frame and these hypotheses can be represented in Fig. 1. In the large rectangle we find meaningful and meaningless phrases depicted by points which have been divided up into four languages.

Figure 1



The languages S_2 and S_3 overlap so that phrase b exists in both languages S_2, S_3. The dotted arrows show exemplary the three functions *des*, *int*, and φ. The set of all phrases which designate the real entity w is shown as a circle, and the set of all phrase which interpret the construct m is shown as an oval. The intersection of these two sets depict the set of the phrases which 'belong to' a pair $\langle w, m\rangle$. A meaningless phrase is be found in the left-hand language at the bottom.

3 The Triplet Structure of a Concept

In the triplet-approach of Kuznetsov has a concept three *basic parts*. This triple-structure can be fitted to the frame formulated above. We look at a distinct concept C, which contains the three basic parts: a set of phrases, a real entity and a construct.

The first basic part of C is an open set of phrases from the appertaining languages: the *set of phrases* Ph_C. Normally, there exist several phrases in Ph_C for the concept C and these stem usually from different languages (for example **gehen**, **walk**, **aller**...). A concept is almost always expressed by phrases from several languages. It is hard to find concepts which are expressed in the English language by **blue**, **between**, or **utility**, and which *cannot* be expressed in other languages. The set of phrases of a concept is open, because each language changes with time. The set Ph_C of phrases of a concept C is a subset of the ensemble of all phrases Ph from our frame: $Ph_C \subseteq Ph$.

The second basic part of the concept C, which we have called *the real entity* of the concept C 'is' something real. More cannot really be directly said about this part. How this real entity precisely 'looks' like, can be expressed only through language or through direct perception.[15] Already, the 'simplest' difference in a real entity leads necessarily to linguistic descriptions (phrases and sentences). Can a real entity 'be seen' in the singular or plural, masculine or feminine? These differences are anchored deeply in the grammar of a language. They are formulated on the linguistic level, not on the level of reality. We can only indirectly through language discover whether a difference applies to a given situation or not. In various languages real entities can be differentiated linguistically in 'monolithic' things and kinds (sets). This difference, too, is dependent on language for its formulation.[16]

We have had a hard time deciding how to formally notate the real entity that corresponds to the concept C. We could just use a symbol, for example Π_2, for designing the second part of a concept. For strategic reasons we have decided on a set notation, so that the second component Π_2 of a concept C is a set which contains a single element ent_C: $\Pi_2 = \{ent_C\}$. In this way we have on one side a symbolic, direct access route to the real entity ent_C, on the other side, the concept C is a part, in view of set theory, of the frame. In other words, through extensive restrictions the frame can become a concept. With the example **blue**, a reader should look at the sky in order to receive an impression of how the real entity $ent_{\mathbf{blue}}$ 'looks' like. To perceive the entity $ent_{\mathbf{between}}$ we can only guess at a situation where three dots land on a line. It is even more difficult for a person to discover the real entity of the concept of utility. The person should have to look at his inner psychological world and at the

[15] See (Quine 1960).

[16] In the German language the word 'Etwas' (something) is often brought together with the word 'Bereich' (area). When this happens an important decision is also made. The entity is 'seen' as a kind (set, plural).

same time at an object in the real world which could be 'useful'[17] to the person.

The third part of the concept C is a fragment of a knowledge system which can be described in many different ways. We are using in this work the theory of sets with which a large part of the knowledge system can be described in a standardized way. Such a special fragment of the knowledge system we call, as in Sec. 2, a (*set-theoretic*) *construct*. The set-theoretic effect lies in the fact that we can reunite various phrases of a concept, rooted in different languages, which designates the same real entity, in the set apparatus.[18]

The three basic parts of a concept are thus bound together to form a unit by the functions, relationships, and constants introduced in Sec. 2. In general, the phrases bind the real entity and the set-theoretical construct of the concept together to form a unit. How do the phrases accomplish this?

The human beings have the ability to denote an entity using a phrase. When a person feels that an entity is important he can use a more permanent symbol, a kind of name for an entity. In a conversation about cursory entities, the phrase which denotes an entity simply is a linguistic, phonetic utterance. On the other hand, an entity, together with a just-used phrase can 'meld' with a person physically, or spiritually. This can then be stored in the mind or body of a person. How this connection, this melding exactly takes place is in this work not a central point for us. There are several disciplines to describe this process in a more interesting way. It is the outcome of such a process that is important for us and we use set theory as a means to describe this outcome. Man has, somehow, succeeded in binding an entity with a set m, so that the set is, somehow, stored within the individual. The set m is, or can be, rather complex that is why, in order to emphasize the complexity of this connecting process, we name it differently. We call such sets *constructs*. The process which connects a phrase to a construct can be expressed through the interpretation function int. The phrase b interprets thus the construct m. The process of interpretation, which we sketched very roughly and handled in a very abstract manner, could be of course filled out with many descriptions from many different disciplines. As discussed in Sec. 2, phrases that belong to a concept, can be – normally, and in human practice – interpreted in set theory uniquely.

Starting with a frame and the points just mentioned, a concept C contains, firstly, the three basic parts: the set of phrases Ph_C, the real entity ent_C and the construct con_C of the concept C. These first three components of a concept are restrictions of the corresponding components of the frame for concepts. These restrictions, for a given concept C, go so far as to limit the set of real entities out of the frame to a singleton containing exact one real entity ent_C, namely the entity *for this* concept C. The set W becomes thus formally a set containing one element ent_C, $\{ent_C\} \subset W$. In the same manner, the set M of constructs of a frame is restricted to a single construct

[17]In order to avoid the word *utility* in this formulation would result in a somewhat lengthy expression.

[18]The formalism of named sets developed by Burgin describes the same effect (Burgin, 1995). In our approach the names belong to the level of designation.

con_C for the concept C: $\{con_C\} \subset M$. In contrast, the set Ph_C of the phrases of the concept C contains many phrases rooted in different languages: $Ph_C \subset Ph$.

Secondly, the concept C contains three relations which arise from the corresponding components *des*, *int* and φ of the frame, so that domain and codomain of each component is restricted to the basic parts of C: Ph_C, $\{ent_C\}$ and $\{con_C\}$. The designator function *des* is restricted to a function des_C, $des_C : Ph_C \to \{ent_C\}$, so that all phrases from Ph_C designate the entity ent_C. *int* is restricted to the function int_C, $int_C : Ph_C \to \{con_C\}$, which interprets each phrase of Ph_C in a construct con_C. Finally, we restrict the correspondence relation φ to the set $\{ent_C\} \times \{con_C\}$ so that φ becomes φ_C, $\varphi_C \subset \varphi$. Hereby, this relation contains exactly one pair $\langle ent_C, con_C \rangle$, that means $\varphi_C = \{\langle ent_C, con_C \rangle\}$. By φ only one entity ent_C is assigned to the construct con_C.

To simplify, we assume that all phrases of a concept are meaningful: $Ph_C \subseteq B$. That is why it is not necessary to carry along the components $\diamond$ and B of each concept. We could also add the last component of the frame Σ, the languages, to a concept. This would, however, lead to a vicious circle. A phrase which belongs to a concept is, first and foremost, an element of language – however not the other way around. We would like to leave this interesting question, which among other things is associated to the meaning of concepts, open.

Thus a concept C has the form:

(1) $$\langle Ph_C, \{ent_C\}, \{con_C\}, des_C, int_C, \varphi_C \rangle.$$

An important question is whether phrases or features of a concept are *essential* or *unessential*. For example, it is not easy to represent the entity of the concept of red. If we say that the entity designated by **red** is made up of the set of all red things, then we are only partially contented. An entity is normally not identical with a set which interprets the appertaining construct for said entity.[19] If we were to take the entity to which the concept of red would like to access, and designate it with the set of all red things, we would not have come much farther in terms of the contents. In this case we would only describe the entity of a concept by using the concept itself. It does not make much sense to try to create concepts out of nothing. The simple, and effective way is to introduce concepts together with theories at the same time. In a color theory, for example, in physical optics, one can characterize the concept of red without essentially using the concept itself. That means, in many cases, it is open to debate whether a phrase for the entity of C is essential or not. If it is not essential, then such a phrase and the appertaining feature of the concept is not a part the concept C. For example the feature 'green-red' is not essential for the concept of red.

We have the same problem with relational concepts. Several concepts are very difficult to express by verbal phrases. For example, we cannot find a verbal phrase which is essential for the concept usually expressed by **the planet Jupiter**. We can,

[19] The entity designated by **Jupiter** is not identical to a set of just one element.

of course, find verbal phrases which have to do with properties of the planet Jupiter. These phrases are however not essential to the concept. For example, there are the well-known 'wandering red patches' on Jupiter. This phrase has a part which belongs to the verbal phrase **wander** and to the appertaining property. This property of Jupiter, and the appertaining verbal phrase, is, of course interesting but not essential for the concept **Jupiter**, at least not in the scientific knowledge of today. In contrast, the concept of causation which is central in the theory of relativity, is expressed in the English language by several essential phrases. We find for example, nominal phrases, like **causal relation**, **causal relationship** or **cause**, and verbal phrases like **investigate whether the causal relationship is anti-symmetrical** or **test whether transitivity plays a part in causal relationships**. Phrases which were, in any case in the past have been, unessential for this concept were for example **psychical cause of a chain reaction of Uranium atoms** or **the effect of 'this' action lies in 'its' past**.

We formulate the structure of a concept so, that the unessential phrases of the concept can be, in short, left out. The *set of all concepts* from the frame R we will denote by $\mathbf{B}(R)$.

To summarize, a concept has the following structure. First, the user of a concept designates something real: the real entity of the concept. Second, the concept is interpreted in a certain construct and third, a concept can be implicitly interpreted in a real entity such that this entity which is often perceived only in a nebulous way is supplied by a relatively clear structure. The real, complex process of designation and interpretation for a given concept is therefore in general restricted to a process we can see through.

We will describe four examples in more detail. The first example is in the English language expressed by the phrase **the blue sky**. The real entity of this concept is often to be seen by a person here and now. C contains the set Ph_C of phrases {**the blue sky**, **der blaue Himmel**,...}, the set $\{ent_C\}$ and a construct $\{con_C\}$. What the person sees, i.e. how this entity is constituted, can be only described by further phrases and sentences. In many descriptions other persons, certain situations, or other entities become more important. A person can generally, perceive several entities at the same time and denotes them through other phrases, like **earth**, **clouds**, **houses**, **clear**, etc. We could easily fill many pages by further describing this entity in language however without ever completing the description. One cannot say much about the construct of this concept in everyday language. The designator function[20] des_C for the concept of 'the blue sky' denotes an abstract phenomenon perceived constantly by billions of people. The function int_C interprets the phrase by a set of possible patterns of colors, whose inner structure we do not discuss further. Little can be said as to the correspondence relation φ_C as well. What corresponds to the set of possible patterns of colors in reality?[21]

[20] We are leaving out the essential addition 'relative to the English language'.
[21] Already the description of the construct as a color is formulated in a physicalist way.

Another simple example for a concept C can be expressed by the word **tree**. The set of phrases is $\{\mathbf{tree}, \mathbf{Baum}, ...\}$. Here we must arrange a construct with two meanings. On one side, it contains a set of tree like things. We will not go into further detail about this 'tree like' structure. On the other side the construct contains a *set of structures* so that each such structure *is* a tree like model. This construct retains in this sense, the double meaning of the concept under consideration. It denotes both an entity which can be seen here and now, and in general an entity which can be 'perceived' for instance as a forest.[22] In our description the correspondence relation can handle both meanings at once.

One of the simplest, scientific concepts is expressed, in the English language, by the word **between**. In the domain of everyday life, we find an endless abundance of aspects and features. If we use this concept in a geometrical sense and limit it to a geometrical theory, the description becomes simple. The concept C contains the set of phrases $\{\mathbf{between}, \mathbf{zwischen}, ...\}$, an entity $ent_{\mathbf{between}}$ and the construct, which is a class of sets, so that each such set – in a given formulation of this geometry – is a ternary relation which fulfills all geometrical axioms of the theory. In such a formulation it must be said, especially, that all objects, which occur in the between-relation, are **points**. It is not very helpful to describe the real entity designated by **between** with the help of normal, English phrases and sentences. The designator function *des* takes the phrase **between**, and 'finds' through this phrase a uniquely determined, real entity. The details must be clarified by the languages. In this example, the word **between** is in logics normally interpreted in a set-theoretical structure. The interpretation of **between** by $int_{\mathbf{between}}$ is thus the class of all sets existing in the above mentioned construct. In other words, in logics, the correspondence relation $\varphi_{\mathbf{between}}$ normally generates a set (a component) of a model. This feature should be extended to all other languages.

A second scientific concept C is expressed by the word **utility**. We are limiting ourselves here to a certain theory in economics, in which this concept is used in a central way.[23] The interpretation of the word through int_C leads to a construct which is a class of sets, so that each such set is a function. This function in turn gives a number, a 'utility-value', which depends on a distinct person and on a distinct good. This function is a component of a structure of a theory in economics. A utility-value is dependent on the requirements and hypotheses made in this theory. Why such a number is called a utility-value, can only be understood when we have internalized the appertaining exchange systems and psychological components. The internalization process is complex. Each person needs much time to accomplish it. The real entity, found through the designator function des_C is also difficult to explain without language. In this example, the concept of utility has a rather theoretical character. In other words, the contents of this concept depends strongly on a theory.[24] In this

[22] The simple solution of double meanings is often decided by the phrase usage. But this way out would complicate our model.

[23] See for example (Balzer et al. 1987), pp. 161.

[24] Interestingly enough, it is not the economic exchange theory, see for example (Balzer, 1985).

case one would read the correspondence relation φ_C in such a way that the clearly describable construct is given and the entity is the corresponding real counterpart. How the phrases of this concept in other important languages (for example in the language of Chinese) will be look like and whether these phrases truly signify the described construct, cannot be answered by us.

4 A Sketch of a Structuralist Analysis of Scientific Concepts

We are now embedding the scientific concepts in the general frame above. As scientific concepts belong to empirical theories, we are using the structuralistic theory of science and are handling the scientific concepts accordingly. In the structuralistic approach, a (*empirical*) *theory* T has the form $\langle K, A, I\rangle$ whereby K is the *formal core* (formulated in set-theoretic terms), A is the *approximation apparatus* and I is a set of *intended applications* (or *intended systems*) for T. The formal core K consists of five classes: M_p, M, M_{pp}, CR and r. M_p is a class of *potential models* and M is a class of *models*: $M \subseteq M_p$. To simplify, we will not be using the other three classes. In addition we will be using a set TM of *special concepts of a theory* T: $K = \langle M_p, M, M_{pp}, CR, r, TM\rangle$. The elements from the classes M and M_p, we call *set-theoretic structures.* A set-theoretic structure y has the form:

$$\langle D_1, ..., D_k, A_1, ..., A_l, R_1, ..., R_m\rangle,$$

where k, l, m are *numbers*, $D_1, ..., D_k$ the *basic sets* of the structure y, $A_1, ..., A_l$ the *auxiliary basic sets* of y and $R_1, ..., R_m$ are the *relations* of y. These sets $D_1, ..., R_m$ we also call *components* (of a set-theoretic structure). The interesting set-theorical structures are those which fulfill the hypotheses for the models and are in addition anchored in an intended system of the theory.

From these components and their elements, which lie in the potential models of the theory, one can form further sets and classes which we call the *terms of the theory* T. Such a term refers to an ensemble of objects, and in set theory the term refers to a set or a class of these objects. When such sets or classes are correctly formed (defined) according to set theory, we have at hand a space of possibilities in which phrases can be interpreted. As an infinite number of terms of a theory can be formed in the set-theoretical apparatus, we need to filter out the more ‘interesting’ terms. First, we define the terms which are always used in the theory, and call them *ground terms* of T. Set-theoretically is a ground term a class of components of potential models of theory T, so that such a component ‘resides’ in a special place of a potential model.

t is a *basic term* (for T) if and only if there is $i \leq k$, so that t is the class of all D_i’s, which are found in each $y \in M_p$ in the i-th place of y, or when there is a $j \leq l$ so that t is the class of all A_j’s, which are found in each $y \in M_p$ in the j-th place of y. In the same way, a *basic relation term* is a class of all relations R_s ($s \leq m$), which can be found in each potential model $y \in M_p$ in the s-th place. Several other, more frequently used terms of a theory can be formed by using set-theoretical definitions or procedures. All these procedures we group together to a set TM (or $TM(T)$) of *special terms* of the theory T, which we have already placed in the core of the theory above.[25] Besides the special terms, we also introduce a set $\mathcal{IR}$ of *inter-theoretical relations*, which is described in structuralist literature.[26]

After these preparations we can add three other components to our frame for concepts: The first component ii the set of scientific concepts $\mathcal{C}$, the second, the set $\mathcal{T}$ of empirical theories, and the third, the set $\mathcal{IR}$ of inter-theoretical relations between these theories.[27]

We fill now the concepts as represented in (1) above with more contents. The structure of a concept gets linked to other components of the frame. We formulate four hypotheses which bind together concepts and theories in the general frame for concepts. We call this entire system made up of concepts and theories, a *model of the theory of scientific concepts*.

Relative to a set of concepts $\mathcal{C}$ we first define, that C is *a concept for* T if and only if there is a special term t of T, so that (1) C is a concept from $\mathcal{C}$, (2) t is a special term of $TM(T)$ and so that (3) the construct of C is a subset of t. Formally:

C is a concept of T iff $\exists t \exists con(C \in \mathcal{C} \wedge \pi_3(C) = con \subseteq t \in TM(T))$.

We can now define a model of the theory of scientific concepts.

The set-theoretical structure x is a *model of the theory of scientific concepts* (SCT) iff there are

$Ph, W, M, des, int, \varphi, B, \diamond, \Sigma, \mathcal{C}, \mathcal{T}, \mathcal{IR}$, so that the following is valid:

1) x has the form $\langle Ph, W, M, des, int, \varphi, B, \diamond, \Sigma, \mathcal{C}, \mathcal{T}, \mathcal{IR}\rangle$
2) $\langle Ph, W, M, des, int, \varphi, B, \diamond, \Sigma\rangle$ is a frame for concepts
3) $\mathcal{C}$ is a set of concepts from $\langle Ph, W, M, des, int, \varphi, B, \diamond, \Sigma\rangle$, i.e. $\mathcal{C} \subseteq \mathbf{B}(\langle Ph, W, M, des, int, \varphi, B, \diamond, \Sigma\rangle)$
4) $\mathcal{T}$ is a non-empty set of *empirical theories*
5) $\mathcal{IR}$ is a set of *inter-theoretical relations* between theories from $\mathcal{T}$

H1) for all theories $T \in \mathcal{T}$ and terms $t \in TM(T)$ there is a concept C, so that the construct con_C of C is a part of the term t, ($con_C \subseteq t$)

H2) for all concepts $C \in \mathcal{C}$ there is a theory $T \in \mathcal{T}$, so that C is a concept[28] of T

H3) for every two languages $S, S' \in \Sigma$ there exists a concept $C \in \mathcal{C}$ and a

[25]The formal details are found for example in (Balzer 1985) and (Balzer et al. 1993). Normally, terms are being considered as designators which are found at the levels of phrases. However, these two meanings of terms can be 'translated' one-to-one. An interesting kind of *inner terms* is used in (Bourbaki 2004).

[26]For example (Balzer et al. 1987), Chap. VI. Instead of the concept 'inter-theoretical relation', one could also use the more general term 'link'.

[27]Both the theories and the concepts contain sets that cannot be traced further back: the set of intended applications and the set of real entities.

[28]I.e. there exist $t \in TM(T)$, so that $con_C \subseteq t \in TM(T)$.

phrase b from the set of phrases Ph_C of C, so that b is an element of both languages S and S'

H4) there exist a set Θ of terms, a function ξ and a function Γ, so that the following holds:

H4.1) ξ assigns for every pair $\langle T, C\rangle$ of theories and concepts, a term $t \in \Theta$

H4.2) Γ assigns to every pair of theories $\langle T, T'\rangle$ a function $\Gamma(T, T')$, so that $\Gamma(T, T') : Ph \to Ph$

H4.3) for every two theories T, T', with are linked by an inter-theoretical relation $\varrho \in \mathcal{IR}$ ($\langle T, T'\rangle \in \varrho$), and for all concepts C, C', all terms t, t' and all phrases b, b', it holds that:
if C is a concept of T and C' is a concept of T',
if t is a term of T and t' is a term of T', ($t \in TM(T), t' \in TM(T')$),
if $\xi(T, C) = t$ and $\xi(T', C') = t'$ and,
if $int(b)$ is the construct[29] con_C of C and
$int(b')$ is the construct $con_{C'}$ of C',
then it holds that $\Gamma(T, T')(b) = b'$.

Hypothesis H1 is not trivial. It would be easy to add a term which does not have an own word. Such a term would be a set-theoretically defined term t, which however would not be a special term of the theory T ($t \notin TM(T)$). Another possibility arises when an entity and the appertaining space of possibilities have been already studied yet do not belong to a term.

Also hypothesis H2 only appears to be simple. For example, it is not easy to find a set-theoretical term to express a concept which in arts and humanities is referred to the word **dialectics**. In non-scientific areas there are of course concepts which cannot be assigned to any of the theories.

The hypothesis H3 expresses informally, that any two languages uses at least one common concept. In other words there is a concept in both languages which can be expressed through the same phrase. For example, the phrase **Windows** denotes at the moment the same thing in each language that uses roman letters.

The last hypothesis H4 remains rather complex, despite preparations to define it. If we decided to stay informal and somewhat vague and use the concept of net, which we have oppressed for simplicity, then H4 could be expressed in the following way. The inter-theoretical relation ρ forms a net of theories, so that on the one hand the terms, concepts and constructs are part of such a net, and so that on the other hand the interpreted phrases ($int(b)$) have found their 'rightful' place in the net.[30] The hypothesis now conveys the following: when two phrases b, b' are 'correctly' joined through the theoretical net, in other words their constructs are similar, both phrases

[29]We use here a 'pure' set theory, which has no *ur*-elements. When we replace '$\subseteq$' by '$\in$', there are constructs which are no sets but are *ur*-elements.

[30]The net of empirical theories, together with the close meshed nets of terms of those theories, is in other words similar to the net of scientific concepts and the appertaining phrases. A formal examination of those networks and their formations is to be found in (Balzer and Sneed 1977, 1978), Sec. IV.

fit together also at the level of designation, i.e. they can be ‘translated’.

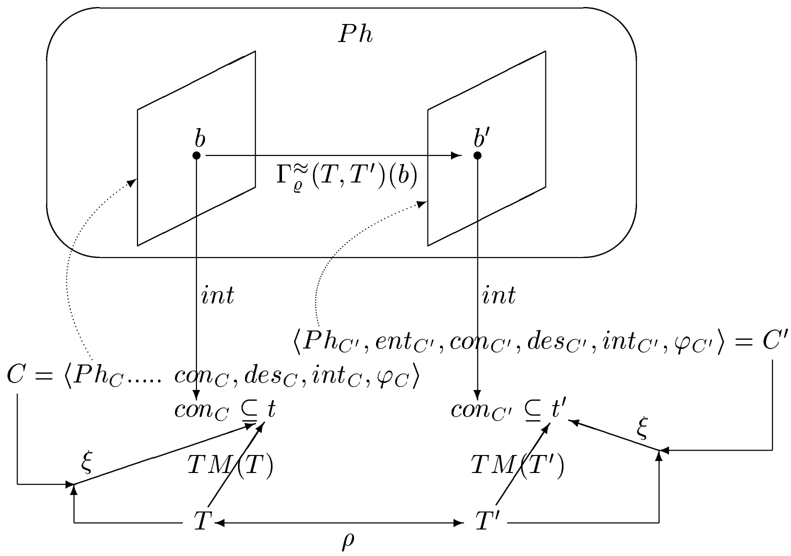
In Fig. 2 we represented a purely local relationship between two phrases and the appertaining concepts, constructs, and theories. In the upper part we find two phrases b, b', which lead through the interpretation function int to the constructs con_C, $con_{C'}$ of the concepts C, C'. The two rectangles form the sets Ph_C, $Ph_{C'}$ of phrases which belong to the concepts C and C'. In the lower part two ‘appertaining’ theories T, T' and two ‘appertaining’ terms t, t' are depicted. The function ξ assigns the left theory T and the concept C to the term t. The same goes for the theory T' and the concept C' on the right. Both terms can be found in the sets $TM(T)$, $TM(T')$ of special terms of these theories. On the left side, the phrase b interprets the construct con_C which belongs to concept C. At this central point, the construct is brought together with the special term t. This is achieved simply by the subset relation $\subseteq$. This link is the ‘hinge’ with which the construct and the term can move. We see the same thing on the right side for $b', C', con_{C'}, t', T'$.

In this situation we require that the phrases b and b' fit together approximately, in short: $\Gamma^{\approx}_{\rho}(T,T')(b) = b'$. This means that a ‘translation function’ $\Gamma(T,T')$ sends the phrase b to the phrase $\Gamma^{\approx}_{\rho}(T,T')(b)$. The function $\Gamma(T,T')$ is formulated in such a way that every (or approximately every) phrase of Ph_C is ‘translated’.[31] The function ξ represents a kind of dictionary, from which one can choose the ‘right’ terms. This function is normally not dependent on the inter-theoretical relation ρ. In contrast, the function Γ is sensitive in response to the given theories T, T' as well as in response to the inter-theoretical relation ρ. This is why we use two indices expressing that Γ is dependent on ρ and can be used in approximation.[32]

Figure 2

[31]For technical reasons all other phrases also get images. The most of these images do not have contents; they come from meaningless phrases.

[32]The topic of approximation is described for example in (Balzer et al. 1987), Chap. 7.



5 Kinds of Concepts

In the general frame, there are of course many sub-categories of concepts, which we cannot systematically describe here. We will only sketch three systems of concepts very roughly. All of these systems of concepts are open systems. Subconcepts, generic concepts, and other kinds of concepts can, over time, fade away, whereas others can be newly formed.

A first system of concepts is based on the Indo-European languages. In this system exist the *concepts for objects*, the *concepts for features* (or *properties*), and the *concepts for relations.* For a *concept C for objects*, the entity ent_C of the concept C is normally a thing, an object or a fact. These distinctions cannot be clearly defined in an ontological sense. The construct of a concept for objects is *not*, under 'normal' circumstances, divided into smaller parts. The phrase **the planet Jupiter** normally designates a single material object, whereas **the crossing of the Red Sea** is a religious event which is not really material in any way.

The construct of a concept for features[33] is normally seen as a set of things. Here too, there are not clear cut boundaries to sort things out. The things found within the borders belong to the concept, the other things are excluded by the concept. Some things just fail to have a special attribute because it is unessential for a certain given feature. For example, the feature of being abstract cannot be attributed to material things. The concept expressed by the word **blue** contains a construct which at first glance, is described circularly by 'the set of all blue things'. When we take a closer

[33] *Natural kinds* are found here.

look it will be revealed that a detailed construct contains other sets which are laid down by quantum mechanics and/or by physiological theories. Another example is **planets** (plural). The construct of this concept, contains in first approximation the set of planets. In a better approximation 'the' set of planets would be described by using physical theories. This formulation too, does not deplete the construct of this concept. The circular description of the contents, which appeared at first glance, can of course be avoided in the same way as with the concept of blue and of red mentioned above.

The construct of a concept for relations is a set of sequences of things, which are linguistically brought into the 'right' order. The construct, for example, of the concept of 'is bigger than', contains – among other things – pairs of things, where the 'one' thing is bigger than the 'other' one and both things are seen in the 'right' order. For the concept expressed by the word **between**, we can describe the designated entity by the rather long phrase **all three items lie on a straight line**.

A second system of concepts is used in logics and set theory, where *all* concepts can be understood as relations. The concepts for objects, features and relations of a natural language system can all be represented by sets. Seen from set theory, all constructs are sets and seen from logics, all constructs are relations. But in set theory a relation is always a set. In this second system, the relations are grouped syntactically by making explicit the types of the relations. The type of a relation expresses that the relation has a distinct number of arguments and a maximum level. A 'natural' relation (a 'real' relationship) has two or more arguments and has the level 1. A concept of this kind is a concept for relations. The examples of **is bigger than** and of **between** were discussed above; the first relation has two arguments, the second has three. An extreme, binary concept for a relation, found in almost all languages, is conveyed by the symbol '=' and is in the English language expressed by the phrase **set-theoretical equality**. A concept of level two is expressed by the phrase **probability function**. The construct consists of the class of all probability functions. Such a function contains among other things a *set* of 'events'. Relations with just one argument (described by **blue**, **planets** or **number of kinds of goods**) belong to the concepts for features. In this system the concepts for objects are handled as limit cases, whereby these concepts have zero arguments. These concepts are often called constants. For example, in a Copernican system, depending on the reconstruction, it is possible to use the phrase **the center of a system of planets** as a constant which designates an untaken point around which the planets circle. In a political state theory, the phrase **the president** is used as a constant.

In this second system of concepts plays the difference between 'pure' relations and functions an important role which can be expanded upon to the other kinds of concepts. A function has two additional properties. A function has (at least) two arguments, and one of the arguments of the function is uniquely determined by the other arguments. For the **utility function**, for example, it is often the case that the number of arguments depends on the number of goods.[34] In such cases the utility function can often have a large number of arguments. Often the constants are used as functions with zero arguments.[35]

The third system of concepts stems from the theory of science. Several concepts associated with a scientific theory T were discussed in the last section. Without much trouble we can divide the concepts for a theory into three important kinds, in which the constructs can be described without approximation. The first subkind contains the *concepts for the basic sets of a theory T*. For the concept C for a basic set of T the construct C is completely identical with the class of all basic sets which are found in the models of T.

In classic particle mechanics CPM there is for example a concept for a basic set of CPM which is expressed – among other things – by the word **particles**. The construct of this concept is the class of sets of particles (things, objects) which appear in the models of CPM. In the economic theory of exchange,[36] PEE, we find a concept for the basic set of PEE which is expressed in the English language by the words **persons** or **actors**. The construct of this concept is the class of the sets of persons, which appears in the models of PEE. The concepts for the basic sets of a theory are, regarding contents, similar to the concepts for features and to the concepts for objects.

The designator function for the phrases works in all these cases in a similar way. In the first case a phrase designates a set of sets of things, in the second case a set of things, and in a third case a thing (objects) – where all these things have the same type. The differences are only based on the different levels of abstraction. The concept expressed by the phrase **the planet Jupiter** designates a concrete object, **the planets** designates a set[37] and **the set of particles of** CPM a set of sets of stars.

A second kind of scientific-theoretical concepts contains *concepts for the auxiliary basic sets of a theory T*. In many examples all the auxiliary basic sets in the models of T are identical. 'The' auxiliary basic set of T is often the set of real or natural numbers. In CPM for example, one of the concepts for an auxiliary basic set of CPM is expressed by the phrase **points of time** (plural). The construct of this concept is a class of sets where each set contains just the points of time which exist in a model of CPM. Analogously, another concept of an auxiliary basic set of PEE is expressed by the phrase **kinds of goods**. In both these examples it is easy to see that the 'auxiliary status' of the auxiliary basic sets are dependent, first of all, on the theory so reconstructed. In CPM the set of points of time from a model are *identified* with the set of real numbers and in PEE the sets of kinds of goods are subsets of the set of natural numbers. These identifications have an auxiliary character that normally

[34] See for example in (Balzer et al. 1987), pp. 161.

[35] See, for example, (Shoenfield 1967), p. 10.

[36] See for example the structuralist reconstructions of the classical particle mechanics CPM and the pure exchange economics PEE in (Balzer et al. 1987), pp. 103 and pp. 161.

[37] Depending on the special formulation of a set theory, these sets can be genuine classes.

is used to simplify longer parts of the construction of a given theory. A point of time 'is' not a real number of course, rather a structure made up of many other objects. The same goes for a kind of goods in PEE.

A concept of the third kind we call a *concept for the basic relations of* T if and only if there exists a term t for a basic relation term of $TM(T)$ so that the construct of the concept is a class of elements of term t. In CPM there is – among other things – a concept for the basic relation which is expressed by the phrase **position function**. A basic relation from a model of CPM is a function which assigns for each particle and each point of time a 'position', and these positions are just handled by mathematical entities (real vectors). The construct of this concept is thus the class of all position functions which can be found in the models of CPM. In PEE we find, for example, the basic relations expressed by the word **utility**, as discussed above.

These three special kinds of concepts, the phrases for basic sets, auxliliary basic sets and for the basic relations of T, are found in all theories T.

Each kind of concepts discussed until now is simply a set of concepts. Such a set has an inner structure which expresses a property (or properties) of the concepts from this set. A concept from one of these kinds is, in other words, a concept for features. Alongside these rather more simple kinds, we would like to take also a closer look to three more complex kinds of concepts. These concepts are, meta-theoretically speaking, genuine relations. In other words, such a concept C can be only determined by a relationship with another concept C' (or with several concepts $C_1 ..., C_n$).

In Carnap (1966) two kinds of concepts are introduced, relative to a given language: the concepts for the observational level and for the theoretical level (of a given language). He distinguishes, in other words, between observational and theoretical concepts in a language. There the observational concepts are independent of the theoretical concepts, in other words, the observational concepts are concepts for features. We will not go into details about the property *observable* of a concept here. The theoretical concepts, in contrast, are concepts for relations. A concept is theoretical only when it can be determined by clear, given hypotheses ('bridge principles') from the given language, which use only the observational concepts.

Sneed (1971) distinguished two other similar kinds: the T-theoretical and the T-non-theoretical concepts of a theory. A T-theoretical concept finds itself in a loose relationship with the T-non-theoretical concepts expressed through the 'second level' sentence, which states the empirical claim of the theory T. In this approach every kind of concept is essential for the other.

There are still more general relationships between scientific-theoretical concepts, that cannot simply stated by the just used distinction of the two kinds. In the structuralistic theory of science these relationships are called *inter-theoretical relations* or *links*. Concepts from several, at least two, theories are linked through inter-theoretical relations. Stated briefly, there is an inter-theoretical relation ρ and there are (at least two) terms C, C' from the theories T, T' so that ρ can be formulated with the concepts C and C' as well as, potentially, with other concepts. The concept from Dalton's stoichiometry $DSTOI$ relates the concept of substance with the concept of body from the rigid body mechanics[38] RBM through a link which uses other concepts of both theories, as well. At the moment there is no wide-spread expression which is used for this general kind of concepts. We shall call these concepts *concepts for links.*[39]

The distinctions between the concepts for links and other kinds of concepts can be further studied within our frame. At this point, we reach a natural boundary at which concepts must work together with other concepts: the concepts are united (again?) with theories.

6 Other Approaches

We sketch out very briefly how several components of earlier approaches can be integrated in our model.

The theory from ancient times about concepts has been actualized for 150 years ago in biology and today through the internet applications in computer science. This theory classifies phrases into categories and other classes. In Sec. 5 we discussed some categories and kinds of concepts of today in our frame.

The components: extent, contents, extension and intension of a concept were introduced in the middle ages and much discussed. The extent (and extension) of a concept which at that time, was viewed as a set of 'things' has further differentiated itself in our model. The easiest way to explain this is through the construct of a concept. A construct is made up of a set of 'elements', which, depending on idealization, can be more or less complex. If we do not analyze these elements further, we come back to the original approach in which the extent of a concept is just a set of things. In our model, we do not have only the construct (here a set) but we use also the real entity which is connected with the extent through the correspondence relation.

Also, the contents and intension of a concept can be best understood through the construct. We are starting with a construct which is in a first step a set. This set can be characterized by a formula which was generated by a sentence. This formula represents then the contents of the concept. The characterization of the set – and thereby the construct – can go 'deep'. The elements of the set are usually quite complex in and of themselves. They are partial constructs which in turn can be characterized through other formulas. We render prominent just one case, namely the concept of utility (see Sec. 3). In this case the construct is a class of utility functions. The characterization of such a utility function, and therefore the contents of this concept, can be briefly formulated as: all the hypotheses of 'a' model of a theory of economics are valid for the utility function. In this example one can formulate the intension of the concept of utility by reference to the construct as follows. A utility function from the construct stems from a (possible) model, to which a (potential) person refers. In each situation

[38] See for example (Balzer et al. 1987), pp. 122.

[39] In the IT- and the cognitive sciences also the phrase **methods** is used. We think that this phrase implies opaque aspects in these applications, and are therefore not using it.

the person can use a different utility function. In another example the color **blue** would be 'seen' differently in different situations.

In our model the contents and the intension of a concept are determined not only by the construct. Contents and intension link the construct also to the real entity and to the phrases of the concept.

The materialistic-dialetical point of view, expressed in, for example, Chupahkin (1973), is only implicitly found in the triplet model. We can however imbed this point of view in our model. The set-theoretical structure of the construct of a concept must, from the materialistic-dialectical viewpoint, have a material basis. This means, the construct must at the end contain also material objects. In addition to that the construct can also have links to many other constructs. Structurally however, these links have to be conveyed through material objects. The dialectical changes of concepts cannot of course be shown in our static model.

Being introduced in the Middle Ages intension leads to differentiations of meaning and sense, Frege (1892). At the end of 19th century, set theory was just being formed, with which it was possible to express such difference very clearly. Seen from set theory, the contents of a concept is limited to the extensional part. In other words, the contents can only be expressed by sentences and formulas, which are in turn formulated in a set-theoretical language. The sense of a concept however is dependent on the situation it finds itself in. The classic example is the concept of 'the planet Venus', which depending on the situation, is called the morning star or the evening star. In our model, the meaning of a concept plays a central role. This is represented by the designator function. Thus, the sense of a concept can be established in our model within the constructs.

A little later, the semantics of possible worlds developed whereby the sense (intension) of a concept could be represented by a function. Such a function assigns to each model, and to each component which could be a part of the model, a purely extensional, descriptive meaning. Instead of 'model' one can, more generally, say 'situation' or 'possible world'.[40] In our model we can place the 'possible worlds function' in the construct of a concept.

Another approach to concepts, where Kuhn (1974) left a lasting impression, contains a new historical-dynamic component. A concept changes with time. In our notation, is the construct of a concept simple – at least in the beginning. The construct is made up of a few paradigmatic examples which of course must be formulated in the language of set theory. When a concept proves useful in a group or in a community, it is then more precisely formulated. That means that the construct receives additions; the concept becomes more complex. This can also be seen by the addition of new phrases, which interpret this enhanced construct. In our model, we can already change the new components in the construct. We cannot however, describe this chronologically. Our static model lacks a time component. Kuhn's approach led to a new description of theory dynamics. A changed theory can be evoked through a

[40] See (Carnap 1947).

revision or a genuine revolution. This important point can too, only be described with a dynamically expanded model. The expansion of a construct functions at the level of phrases so that by the correspondence relation also the real entity can be changed whether marginally or radically. This is the point where the philosophical directions diverge. A realist insists on the unchangeability of real entities, whereas a member of a different school of thought, a pragmatic for example, views this point in a more relaxed manner.

In Putnam (1975) a similar approach is followed. It contains more logical and linguistic aspects and works with stereotypes rather than paradigms. There, the extent (*reference*) is an independent component of the concepts where in our work, it contains additional aspects discussed above. Putnam uses syntactical and semantical components for a concept which are often dependent on a language or a language family. In our more general, linguistically comprehensive model we can accommodate syntactic and semantic components in a better way. Several elements from the construct can, depending on the languages given within our frame, be put into perspective.

The newest studies on concepts are coming from pragmatics, where internet files are combed through to generate concepts by means of different computer methods. In this field, several computer methods are being invented (or re-invented) and tested, often for very particular, practical purposes.[41] These methods are in principle classificatory. The lists or files, which are formed in the first stage, can today be automatically classified or linked to other lists or files. At this point, we did not yet try to incorporate these methods in our model or in an expanded version of our model. These computer methods however, are not really linked to earlier, more formal methods. Apparentely, computer methods find themselves in a rather simple state where 'the wheel is just re-invented'.

Bibliography

Andersen, H., Barker, P. & Chen X. (2006). *The Cognitive Structure of Scientific Revolutions.* New York: Cambridge University Press.

Balzer, W. (1985). *Theorie und Messung.* Berlin Heidelberg: Springer.

Balzer, W. & Sneed, J.D. (1977). Generalized Net Structures of Empirical Theories, I. *Studia Logica, XXXVI*, 195-211.

Balzer, W. & Sneed, J.D. (1978). Generalized Net Structures of Empirical Theories, II. *Studia Logica, XXXVII*, 167-94.

Balzer, W., Moulines, C.U. & Sneed, J.D. (1987). *An architectonic for science.* Dorrecht: D. Reidel Publishing Company.

Balzer, W., Lauth, B. & Zoubek, G. (1993). A model for science kinematics. *Studia Logica, 52*, 519-48.

Bourbaki, N. (2004). *Theory of sets.* Berlin Heidelberg: Springer-Verlag. 1. ed. 1968.

[41] For example (Dahlberg 1981; Cooper 1994).

Bünting, K.-D. (1984). *Einführung in die Linguistik* (15.Aufl.). Weinheim: Beltz Athenäum Verlag.

Burgin, M. (1995). Named sets as a basic tool in epistemology. *Epistemology, 18*, 87-100.

Burgin, M. & Kuznetsov, V. (1994). Scientific Problems and Questions From a Logical Point of View. *Synthese, 100*, 1-28.

Carnap, R. (1947). *Meaning and necessity. A study in semantic and modal logic.* Chicago: University Chicago Press.

Carnap, R. (1966). In Gardner M. (Ed.) *Philosophical foundations of physics.* New York: Basic Books.

Chupahkin, I. Y. (1973). *Methodological questions of theory of concepts.* Leningrad: Leningrad University. (in Russian language).

Cohen B. & Murphy, G.L. (1984). Models of concepts, *Cognitive Science, 8*, 27-58.

Cooper, M. W. (1994). Is medicine hermeneutics all the way down?. *Theoretical Medicine and Bioethics, 15*, 149 - 180.

Dahlberg, I. (1981). Conceptual definitions for INTERCONCEPT, *International Classification, 8*, 16 - 22.

Diederich, W., Ibarra, A. & Mormann, T. (1989). Bibliography of structuralism 1971-1988. *Erkenntnis, 30*, 387-407.

Diederich, W., Ibarra, A. & Mormann, T. (1994). Bibliography of structuralism II 1989-1994 and additions. *Erkenntnis, 41*, 403-418.

Diez, J. A. (2002). A program for the individuation of scientific concepts. *Synthese, 130*, 13-48.

Fodor, J. (1998). *Concepts. Where cognitive science went wrong.* Oxford: Oxford University Press.

Frege, G. (1892). Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik, NF 100*, 25-50.

Gabora, G. & Aerts, D. (2002). Contextualizing concepts using a mathematical generalization of the quantum formalism. *Journal of Experimental and Theoretical Artificial Intelligence, 14*, 327-358.

Gleitman L. R. & Gleitman, H. (2007). Why stereotypes don't even make good defaults. *Cognition, 103*, 1-22.

Gorskiy, D. P. (1983). Concept. *Philosophical Encyclopedical Dictionary, 1983*, 513-514. Moskau. (in Russian language).

Hjorland, B. (2009). Concept theory. *Journal of the American Society for Information Science and Technology, 60*(8), 1519 - 1536.

Kuhn, T. S. (1974). Second thoughts on paradigms. In Suppe F. (Ed.) *The structure of scientific theories* (pp. 459 - 482). Urbana: University of Illinois Press.

Kuznetsov, V. (1994). Types of fuzzy concepts. In *Proceedings of the second european congress on intelligent techniques and soft computing* (Vol.2, pp. 675-79). Aachen.

Kuznetsov, V. (1996a). Theory and concept metamorphoses, Cogita humana – Dynamik des Wissens und der Werte. XVII Deutscher Kongreß für Philosophie, C. Hübig

and H.Poser (eds.), workshop-papers, Vol 1. Allgemeine Gesellschaft für Philosophie in Deutschland and Institut für Philosophie der Universität Leipzig, 568-75.
Kuznetsov, V. (1996b). On triplet classification of concepts. *Knowledge Organization, 24*, 163-75.
Kuznetsov, V. (1997). *A Concept and its structures: The methodological analysis* (in Russian language). Kiev: Institute of Philosophy of National Academy of Sciences of Ukraine. ISBN 966-7234-05-3.
Kuznetsov, V. & Kuznetsova, E. (1998). Types of concept fuzziness. *Fuzzy Set and Systems, 96*, 129-38.
Kuznetsov, V. (1999). On the triplet frame of concept analysis. *Theoria, 14*, 39-62.
Kuznetsov, V. (2002). The triplet modeling of representing relations between object concepts. In *Proceedings of the twelfth European-Japanese conference on information modeling and knowledge basis*, Krippen, Germany (pp. 14-23).
Kuznetsov, V. (2003). The triplet modeling of concept connections. In A. Rojszczak, J. Cachro & G. Kurczewski (Eds.), *Philosophical dimensions of logic and science*, selected contributed papers from the eleventh international congress of logic, methodology, and philosophy of science (pp. 317-30). Dordrecht: D. Reidel Publishing Company.
Kuznetsov, V. (2004). On representing relations between physical concepts. *Communications and Cognition, 37*, 105-35.
Lorenzen, P. (1987). *Lehrbuch der konstruktiven Wissenschaftstheorie.* Mannheim: Bibliographisches Institut.
Murphy, G. (2002). *The big book of concepts.* Cambridge MA: MIT Press.
Peacock, C. (1992). *A study of concepts.* Cambridge: MIT Press.
Prinz, J. (2002). *Furnishing the mind: Concepts and their perceptual basis.* Cambridge MA: MIT Press.
Putnam, H. (1975). The Meaning of 'Meaning'. In Putnam, H. (Ed.), *Mind, language and reality. Philosophical papers* (vol.2, pp. 215-272). Cambridge: Harvard University Press.
Quine, W. v. O. (1960). *Word and object.* Cambridge MA: MIT Press.
Radford, A. (1981). *Transformational syntax.* Cambridge: Harvard University Press.
Ros, A. (1989). *Begründung und Begriff. Wandlungen des Verständnisses begrifflicher Argumentationen.* Vol. I, Hamburg: Meiner Verlag.
Ros, A. (1990a). *Begründung und Begriff. Wandlungen des Verständnisses begrifflicher Argumentationen.* Vol. II, Hamburg: Meiner Verlag.
Ros, A. (1990b). *Begründung und Begriff. Wandlungen des Verständnisses begrifflicher Argumentationen.* Vol. III, Hamburg: Meiner Verlag.
Shoenfield, J.R. (1967). *Mathematical logic.* Reading MA: Addison-Wesley Publishing Company.
Sneed, J. D. (1971). *The logical structure of mathematical physics.* Dordrecht: D. Reidel Publishing Company. (2nd ed. 1991).
Wachman, M. (2000). A model of concept formation. *Mathematical and Computer*

Modelling, 32, 1209-1286.
Weitz, M. (1988). *Theories of concepts: A history of the major philosophical tradition.* New York: Routledge.